

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT2

Branch : Diploma(All)

Semester : 2 **Date :** 06/05/2016

Time : 10:30 To 1:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

- a) If the two vectors \vec{x} and \vec{y} are perpendicular to each other then $\vec{x} \cdot \vec{y} = \underline{\hspace{2cm}}$.
(a) 0 (b) 1 (c) -1 (d) none of these
- b) If $\vec{a} = 2i + 3j$ and $\vec{b} = 3i - j - 2k$ then $\vec{a} - \vec{b} = \underline{\hspace{2cm}}$.
(a) (-1, 4, -2) (b) (-1, 4, 2) (c) (1, 4, -2) (d) none of these
- c) $|(2, 1, 1) + (1, 2, 3)| = \underline{\hspace{2cm}}$
(a) $\sqrt{17}$ (b) $2\sqrt{17}$ (c) 34 (d) $\sqrt{34}$
- d) If $f(x) = \sin x$ then $f(2\pi) = \underline{\hspace{2cm}}$
(a) -1 (b) 0 (c) 1 (d) 2
- e) $\lim_{x \rightarrow 0} \left(1 + \frac{3}{x}\right)^x = \underline{\hspace{2cm}}$
(a) e^3 (b) e (c) $e^{1/3}$ (d) none of these
- f) $\lim_{x \rightarrow 0} \frac{x^n - a^n}{x - a} = \underline{\hspace{2cm}}$
(a) ax^{n-1} (b) nx^{n-1} (c) na^{n-1} (d) none of these
- g) $\lim_{x \rightarrow 0} \frac{x^2 + x + 1}{x + 1} = \underline{\hspace{2cm}}$
(a) 1 (b) 0 (c) -1 (d) none of these
- h) $\frac{d\left(\frac{1}{x}\right)}{dx} = \underline{\hspace{2cm}}$
(a) $\log x$ (b) $-\frac{1}{x}$ (c) $\frac{1}{x^2}$ (d) $-\frac{1}{x^2}$



- i) $\frac{d(\tan x)}{dx} = \underline{\hspace{2cm}}$
 (a) $\operatorname{cosec}^2 x$ (b) $-\operatorname{cosec}^2 x$ (c) $\sec^2 x$ (d) $-\sec^2 x$
- j) $\frac{d(\sin^{-1} x)}{dx} = \underline{\hspace{2cm}}$
 (a) $\frac{-1}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{1+x^2}$ (d) none of these
- k) $\int a^x dx = \underline{\hspace{2cm}}$
 (a) $a^x \log_e a + c$ (b) $\frac{\log_e a}{a^x} + c$ (c) $\frac{a^x}{\log_e a} + c$ (d) none of these
- l) $\int \cos x dx = \underline{\hspace{2cm}}$
 (a) $\sec x + c$ (b) $-\sec x + c$ (c) $-\sin x + c$ (d) $\sin x + c$
- m) $\int \frac{1}{\sqrt{1-x^2}} dx = \underline{\hspace{2cm}}$
 (a) $\sin^{-1} x + c$ (b) $-\sin^{-1} x + c$ (c) $\cos^{-1} x + c$ (d) $-\cos^{-1} x + c$
- n) $\int_0^1 e^x dx = \underline{\hspace{2cm}}$
 (a) $1-e$ (b) $e-1$ (c) e (d) none of these

Attempt any four questions from Q-2 to Q-8

- Q-2 Attempt all questions (14)**
- a) If $\bar{a} = i + 2j - k$, $\bar{b} = 3i + j + 2k$ and $\bar{c} = -2i - j + 5k$ then find $|2\bar{a} + 3\bar{b} - \bar{c}|$. (5)
- b) Evaluate: $\lim_{x \rightarrow a} \frac{\sqrt{2a-x} - \sqrt{x}}{a-x}$ (5)
- c) Find $\frac{dy}{dx}$ if $y = \frac{1 + \tan x}{1 - \tan x}$. (4)
- Q-3 Attempt all questions (14)**
- a) Prove that the angle between two vectors $3i + j + 2k$ and $2i - 2j + 4k$ is $\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$. (5)
- b) Using definition find derivative of $f(x) = x^3 - 2x$. (5)
- c) Find $\int \frac{\cos \sqrt{x}}{2\sqrt{x}} dx$. (4)
- Q-4 Attempt all questions (14)**
- a) Evaluate: $\lim_{x \rightarrow 0} \frac{3^{2x} - 2^{2x}}{x}$ (5)



b) Find $\frac{dy}{dx}$ if $y = \log\left(\frac{\sin x}{1 + \cos x}\right)$. (5)

c) For what value of R, vectors $2i - 3j + 5k$ and $Ri - 6j - 8k$ are perpendicular to each other? (4)

Q-5

Attempt all questions (14)

a) Find $\frac{dy}{dx}$ at $t = 3$ if $x = \frac{a}{2}\left(t + \frac{1}{t}\right)$ and $y = \frac{b}{2}\left(t - \frac{1}{t}\right)$. (5)

b) Find $\int x \sin x \, dx$. (5)

c) Evaluate: $\lim_{x \rightarrow 2} \frac{x^3 - 3x^2 + 2x}{x^2 - 4}$ (4)

Q-6

Attempt all questions (14)

a) Forces $\vec{F}_1 = i + 2j - 3k$ and $\vec{F}_2 = i - j + 2k$ act on a particle under the influence of these forces, particle moves from the point $(3, 1, 2)$ to $(1, 3, -1)$. Find the work done. (5)

b) Prove that $\int_0^{\pi/2} \frac{\tan x}{\tan x + \cot x} \, dx = \frac{\pi}{4}$. (5)

c) The equation of motion of a particle is $s = -5t^3 + 15t + 3$. Find velocity and acceleration after 3 seconds. (4)

Q-7

Attempt all questions (14)

a) If $f'(x) = 4x^2 + 6x - 3$ and $f(1) = 2$ then find function $f(x)$. (5)

b) Find $\frac{dy}{dx}$ if $y = (\sin x)^x$. (5)

c) Find unit vector which is perpendicular to $\vec{x} = 2i + 3j + 2k$ and $\vec{y} = 5i - 3j + 5k$. (4)

Q-8

Attempt all questions (14)

a) Find area of region bounded between $y = x^2$, X-axis, $x = 1$ and $x = 2$. (5)

b) If $\vec{a} = (2, -3, -1)$ and $\vec{b} = (1, 4, -3)$ then find $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$. (5)

c) If $f(x) = e^x$ then prove that (i) $f(x + y) = f(x)f(y)$ (ii) $f(x - y) = \frac{f(x)}{f(y)}$. (4)

